Table 1 Constant values in Eq. (1)

Equation	K_1	K_2	Region
(1) Boundary layer	0	0	For all Y
(2) Coupled boundary layer/inviscid[a]	0	0	$Y \leq \delta$
	1	0	$Y > \delta$
(3) Thin layer	1	0	For all Y
(4) Navier-Stokes	1	1	For all Y

[a]Also requires $\omega = 0$ for $Y > \delta$.

While conclusions that may be drawn on the basis of these solutions are strictly valid only for incompressible laminar flow, they can be plausibly assumed to apply to a much larger class of flows. We note that the differences between the various approximations and the full equations decrease with increasing Reynolds number as expected. Since most aerodynamically significant flows are at high Reynolds numbers, the error introduced by the approximation to the governing equations will tend to be small and will certainly be insignificant relative to the errors introduced by the turbulence model. Compressibility effects should not alter the present conclusions insofar as viscous effects are concerned, and the direct effects of shock waves and normal pressure gradients within the boundary layer as sources of error should be relatively small for moderate Mach numbers. In this regard, the recent comparisons of Navier-Stokes and thin-layer solutions for supersonic corner flow[6] are informative.

For any flow, or region of flow, for which viscous-inviscid interaction effects are small, classical boundary-layer equations will provide a satisfactory description of the viscous flow at a fraction of the computational cost of any higher approximation. For flows with significant viscous-inviscid interaction, as long as they are boundary-layer-like, i.e., $v/u \ll 1$, coupled boundary-layer/inviscid equations will provide an adequate flow description.

References

[1]Steger, J. L., "Implicit Finite Difference Simulations of Flow About Arbitrary Geometrics with Application to Airfoils," *AIAA Journal*, Vol. 16, July 1978, pp. 679-686.

[2]Le Balleur, J. C., Peyret, R., and Viviand, H., "Numerical Studies in High Reynolds Number Aerodynamics," ONERA TP 1979-99, 1979.

[3]Ghia, R. N., Ghia, U., and Tesch, W. A., "Evaluation of Several Approximate Models for Laminar Incompressible Separation by Comparison with Complete Navier-Stokes Solutions," AGARD CP 168, Conference on Flow Separation, Göttingen, May 1975.

[4]Murphy, J. D., "An Efficient Solution Procedure for the Incompressible Navier-Stokes Equations," *AIAA Journal*, Vol. 15, Sept. 1977, pp. 1307-1314.

[5]Briley, W. R., "A Numerical Study of Laminar Separation Bubbles Using the Navier-Stokes Equations," *Journal of Fluid Mechanics*. Vol. 47, 1971, pp. 713, 736.

[6]Hung, C. M. and Kurasaki, S. S., "Thin Layer Approximation for Three-Dimensional Supersonic Corner Flows," *AIAA Journal*, Vol. 18, Dec. 1980, pp. 1544-1546.

Decomposition of a Disturbance in Parallel Shear Flows

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It is well known[1] that, *locally* in space and time, a small perturbation in a fluid may be decomposed into three types of nearly independent fluctuations: vorticity, pressure, and entropy modes. Roughly speaking, both the vorticity and entropy modes convect with the fluid while inducing solenoidal and irrotational velocity fields, respectively, without significant pressure fluctuations. Indeed, most of the pressure fluctuations are carried by acoustic waves that propagate relative to the fluid at the speed of sound. The velocity field associated with the pressure fluctuations is irrotational. Because of the nonlinearity of the equations of motion, these disturbances interact quadratically whenever their amplitudes are no longer infinitesimal. For example, vorticity-vorticity interactions are responsible for the generation of sound by turbulence whereas sound-sound interactions lead to shock waves in a finite amplitude pressure field.

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While a *local* study of the flow is extremely important and useful, such study cannot provide much information on the *global* behavior of a disturbance except when the disturbance is propagating through a uniform undisturbed state. The reason is that the interaction between the mean flow gradients and the disturbance is ignored.

An important extension of these classical ideas on the decomposition of a flow is given by Goldstein.[2] Goldstein shows that for irrotational base flows, the perturbation velocity $\boldsymbol{u}$ may be decomposed into

$$\boldsymbol{u} = \nabla G + \boldsymbol{v} \tag{1a}$$

where, for incompressible flows,

$$\Delta G = -\nabla \cdot \boldsymbol{v} \tag{1b}$$

Δ is the Laplacian and G the velocity potential. The velocity field $\boldsymbol{v}$, which is called a *generalized convected disturbance*, satisfies

$$\frac{\mathrm{D}\boldsymbol{v}}{\mathrm{D}t} + \boldsymbol{v} \cdot \nabla \boldsymbol{U} = 0 \tag{1c}$$

where $\mathrm{D}/\mathrm{D}t$ denotes the time derivative following the irrotational mean flow velocity $\boldsymbol{U}$, and the perturbation pressure p is given by

$$\frac{p}{\rho} = -\frac{\mathrm{D}G}{\mathrm{D}t} \tag{1d}$$

where $\rho = \text{const}$ is the fluid density. Note that Eqs. (1) are valid for constant density flows without entropy fluctuations. Quite generally, Eq. (1c) can be solved in closed form[2] and the complete perturbation velocity field $\boldsymbol{u}$ may be obtained by solving a single Poisson equation (1b). Perhaps it is worth pointing out that Goldstein's results[2] are valid also for compressible flows and, in that case, significant simplifications may be obtained by making the tangent gas approximation.[3]

It is not possible to extend these ideas in a perfectly satisfactory way to rotational base flows. However, for parallel shear flows, Goldstein[4] has given a valid decomposition. The purpose of this Note is to show that the decomposition is not unique, and an alternate one which resembles Eqs. (1) quite closely can be derived from elementary considerations.

Consider a right-handed Cartesian coordinate system $\boldsymbol{x} = (x, y, z)$, fixed in space, such that the x axis is along the mean flow direction. A *simple* incompressible parallel shear flow is defined by the conditions that the unperturbed pressure and density are uniform and the velocity has components $[U(y), 0, 0]$. It is easy to see that such flow satisfies the equations of motion.

If this flow is perturbed by a disturbance, the linearized equations describing the evolution of the disturbance are

$$\nabla \cdot \boldsymbol{u} = 0 \tag{2a}$$

$$\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t}+\boldsymbol{e}_x v\frac{\mathrm{d}U}{\mathrm{d}y}=-\frac{\nabla p}{\rho} \tag{2b}$$

$$\frac{\mathrm{D}}{\mathrm{D}t}=\frac{\partial}{\partial t}+U\frac{\partial}{\partial x} \tag{2c}$$

where the perturbation velocity components are $\boldsymbol{u}=(u, v, w)$, p denotes the perturbation pressure, $\rho=\text{const}$ is the unperturbed density, the unit vector along the x axis is $\boldsymbol{e}_x$, and t stands for time. The ideas described herein may be applied to more general shear flows in which the streamwise component of the mean velocity U depends on both of the crossplane coordinates (y,z). Such application may be made by a suitable transformation of coordinates, as described by Goldstein.[4] Perhaps it is worthwhile to point out that the parallel flow approximation is used in many branches of fluid mechanics, e.g., hydrodynamic stability[5] or jet noise theory.[6]

It is well known that Eqs. (2) may be rearranged into a single partial differential equation for the perturbation pressure through cross differentiation

$$\frac{\mathrm{D}}{\mathrm{D}t}\Delta p-2\frac{\mathrm{d}U}{\mathrm{d}y}\frac{\partial^2 p}{\partial x\partial y}=0 \tag{3}$$

The crucial observation in this Note is the fact that once the pressure is determined from Eq. (3), the momentum equation (2b) may be regarded as a system of linear, first-order partial differential equations for the velocity components (u,v,w). This system is essentially uncoupled, therefore, readily solved.

Since the governing equations are linear with coefficients independent of time, the time dependence is extracted through a Fourier transform. This is equivalent to writing

$$\boldsymbol{u}(x,t)=\hat{\boldsymbol{u}}(x)\exp(i\omega t),\qquad i=(-1)^{1/2} \tag{4a}$$

$$p(x,t)=\hat{p}(x)\exp(i\omega t) \tag{4b}$$

where $\omega=\text{const}$ may be interpreted as the radian frequency of oscillation. In order to simplify the notation, $(\cdot)$ and $(\hat{\cdot})$ are used interchangeably since it is clear from the context which one is referred to.

Before the solution to Eq. (2b) is written down, the very important concept of a convected disturbance is discussed. Such disturbance δ obeys the equation

$$\frac{\mathrm{D}\delta}{\mathrm{D}t}=0 \tag{5a}$$

whose solution is

$$\hat{\delta}(x)=\delta_0(y,z)\exp(-i\omega x/U) \tag{5b}$$

where δ_0 is an arbitrary function of the cross-stream coordinates. An arbitrary convected disturbance is abbreviated ACD.

From the third component of the momentum equation, it is a simple matter to show

$$w=\mathrm{ADC}+\frac{\partial\phi}{\partial z} \tag{6a}$$

where

$$\phi=\frac{\exp(-i\omega x/U)}{\rho U}\int_0^x p(\alpha,y,z)\exp\left(\frac{i\omega\alpha}{U}\right)\mathrm{d}\alpha \tag{6b}$$

Similarly, from the second and first components of the momentum equations, it follows, after some algebra,

$$v=\mathrm{ACD}+\frac{\partial\phi}{\partial y}+\frac{1}{U}\frac{\mathrm{d}U}{\mathrm{d}y}(1-i\omega x/U)\phi+\frac{i\omega}{U^2}\frac{\mathrm{d}U}{\mathrm{d}y}\psi \tag{6c}$$

$$u=\mathrm{ACD}+\frac{\partial\phi}{\partial x}-\frac{\mathrm{d}U}{\mathrm{d}y}\theta \tag{6d}$$

where

$$\psi=-\frac{\exp(-i\omega x/U)}{\rho U}\int_0^x p(\alpha,y,z)\exp\left(\frac{i\omega\alpha}{U}\right)\alpha\mathrm{d}\alpha \tag{6e}$$

$$\theta=\frac{\exp(-i\omega x/U)}{U}\int_0^x v(\alpha,y,z)\exp\left(\frac{i\omega\alpha}{U}\right)\mathrm{d}\alpha \tag{6f}$$

The "arbitrary" functions ϕ, ψ, and θ are not at all independent; they are all determined by the pressure. Reverting back to the time domain in place of the frequency domain one finds

$$\frac{\mathrm{D}\phi}{\mathrm{D}t}=-\frac{p}{\rho} \tag{7a}$$

$$\frac{\mathrm{D}\psi}{\mathrm{D}t}=x\frac{\mathrm{D}\phi}{\mathrm{D}t} \tag{7b}$$

$$\frac{\mathrm{D}^2\theta}{\mathrm{D}t^2}=\frac{\partial}{\partial y}\frac{\mathrm{D}\phi}{\mathrm{D}t} \tag{7c}$$

and

$$u=\mathrm{ACD}+\frac{\partial\phi}{\partial x}-\frac{\mathrm{d}U}{\mathrm{d}y}\theta \tag{8a}$$

$$v=\mathrm{ACD}+\frac{\partial\phi}{\partial y}-\frac{x}{U^2}\frac{\mathrm{d}U}{\mathrm{d}y}\frac{\partial\phi}{\partial t}+\frac{1}{U}\frac{\mathrm{d}U}{\mathrm{d}y}\phi+\frac{1}{U^2}\frac{\mathrm{d}U}{\mathrm{d}y}\frac{\partial\psi}{\partial t} \tag{8b}$$

$$w=\mathrm{ACD}+\frac{\partial\phi}{\partial z} \tag{8c}$$

Equations (7) and (8) are the central result of this Note. They clearly express the velocity field as the sum of an arbitrary convected disturbance (ACD), the gradient of a velocity potential ϕ, and other important terms which are essentially responsible for the production of perturbation vorticity. These terms are proportional to the mean velocity gradient and are absent in uniform flows. In this decomposition, the pressure [see Eq. (7a)] is still the convective derivative of the velocity potential, as in the case of the more classical decompositions.

On the other hand, Goldstein[4] obtains the following result:

$$u=\frac{\partial}{\partial x}\left(\frac{\mathrm{D}^2\Phi}{\mathrm{D}t^2}\right)+\frac{\mathrm{d}U}{\mathrm{d}y}\left[-\frac{\partial}{\partial y}\frac{\mathrm{D}\Phi}{\mathrm{D}t}+\frac{\partial\Theta}{\partial z}-2\frac{\mathrm{d}U}{\mathrm{d}y}\frac{\partial\Phi}{\partial x}\right] \tag{9a}$$

$$v=\frac{\partial}{\partial y}\left(\frac{\mathrm{D}^2\Phi}{\mathrm{D}t^2}\right)+\frac{\mathrm{d}U}{\mathrm{d}y}\left(\frac{\partial}{\partial x}\frac{\mathrm{D}\Phi}{\mathrm{D}t}\right) \tag{9b}$$

$$w=\frac{\partial}{\partial z}\left(\frac{\mathrm{D}^2\Phi}{\mathrm{D}t^2}\right)+\frac{\mathrm{d}U}{\mathrm{d}y}\left(-\frac{\partial\Theta}{\partial y}\right) \tag{9c}$$

where Θ is an arbitrary convected disturbance and $-p/\rho=\mathrm{D}^3\Phi/\mathrm{D}t^3$.

The decomposition derived herein, Eqs. (8), is more explicit in its dependence on the velocity potential and on an arbitrary convected disturbance than the Goldstein result. Furthermore, the usual connection between the velocity potential and the perturbation pressure is retained; in some way, Eqs. (8) represent a direct generalization of the results valid for irrotational base flows. It should be noted, however, that Eqs. (8) may be converted into Eqs. (9) by suitable rearrangement.

References

[1]Chu, B. T. and Kovasznay, L. G., "Non-Linear Interactions in a Viscous Heat-Conducting Compressible Gas," *Journal of Fluid Mechanics*, Vol. 3, 1958, pp. 494-514.

[2]Goldstein, M. E., "Unsteady Vortical and Entropic Distortions of Potential Flows Round Arbitrary Obstacles," *Journal of Fluid Mechanics*, Vol. 89, 1978, pp. 433-468.

[3]Kerschen, E. J. and Balsa, T. F., "Transformation of the Equations Governing Disturbances of Two-Dimensional Compressible Flow," *AIAA Journal*, Vol. 19, Oct. 1981, pp. 1367-1370.

[4]Goldstein, M. E., "Scattering and Distortion of the Unsteady Motion on Transversely Sheared Mean Flows," *Journal of Fluid Mechanics*, Vol. 91, 1979, pp. 601-632.

[5]Lin, C. C., *The Theory of Hydrodynamic Stability*, Cambridge University Press, London, 1966.

[6]Lilley, G. M., "On the Noise from Jets," AGARD CP-131, 1974.

Transient Decay Times and Mean Values of Unsteady Oscillations in Transonic Flow

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Introduction

IN Ref. 1, Kerlick and Nixon have made the important point that, when using a finite difference, time-marching computer code to investigate the lift on an oscillating airfoil in transonic flow, it is necessary to carry the solution sufficiently far forward in time that an essentially steady-state solution is obtained. Moreover, they offer a method for estimating the transient time before the steady state is reached. They note that, if one stops the time-marching solution before the transient is complete and the steady state is reached, then one may reach the incorrect conclusion that a change in the mean lift has occurred due to the oscillating motion of the airfoil when in fact no such change has occurred.

Nevertheless, what is perhaps surprising, and the principal reason for the present study, is that for a narrow Mach number range the time for the transient to decay and a steady state to be reached is extraordinarily long. Moreover, for a very narrow range of Mach number a nonzero mean value of lift can occur for an airfoil of symmetrical profile oscillating about a zero angle of attack.

Technical Discussion

In Ref. 1 a NACA 64A006 airfoil was studied using the LTRAN2 computer code for a freestream Mach number of 0.875 with the airfoil oscillating at a peak angle of attack of 0.25 deg and with various reduced frequencies k. It was shown that typically up to six cycles of airfoil oscillation must be considered for the mean lift to be less than 1% of the corresponding oscillatory lift peak value. In Ref. 2 the first author and his colleagues (also using the LTRAN2 computer code) examined various airfoils (64A006, 64A010, MBB-A3) at various Mach numbers and reduced frequencies. The calculations in Ref. 2 were carried forward in time through six cycles of airfoil oscillation. As was found by Kerlick and Nixon, the results of Ref. 2 for the (symmetric) 64A006 airfoil showed the mean or average lift with the airfoil oscillating to be essentially unchanged from its value for no airfoil oscillation (i.e., zero) for *most* Mach numbers studied *after six cycles of oscillation*. However at $M_\infty = 0.88$ and particularly at $M_\infty = 0.9$ this was *not* the case. Hence, the first author *incorrectly* concluded that a change in mean or average lift had occurred. The correct conclusion is that at $M_\infty = 0.9$ and 0.88 many cycles of oscillation (>40) are required for the mean lift to decay to essentially zero.

The results presented in Fig. 1 are for $M_\infty = 0.9$, $k = 0.2$, and various peak oscillatory angles of attack.[2] They are for the lift coefficient; similar results (not shown for brevity) were obtained for moment coefficient (about the midchord) and shock displacement (normalized by the airfoil chord). Both mean steady values and first harmonic values are shown *after six cycles of oscillation*. As can be seen, the change in mean lift at this Mach number can be as much as 50% of the peak oscillatory lift. For $M = 0.88$ the mean or average components are never more than 10% of the first harmonic oscillatory components and hence these results are not displayed.

Inspired by the Note of Kerlick and Nixon, the case of $k = 0.2$ and a peak oscillating angle of attack of 0.5 deg was

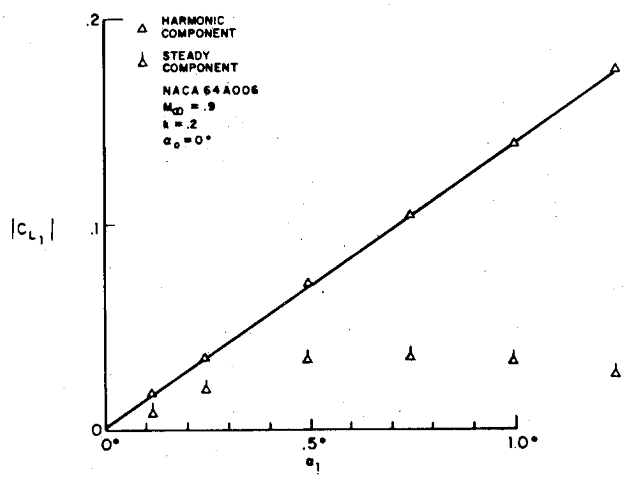

Fig. 1 Steady-state and first harmonic lift components vs dynamic angle of attack, NACA 64A006, $M_\infty = 0.9$, $k = 0.2$, $\alpha_0 = 0$ deg.

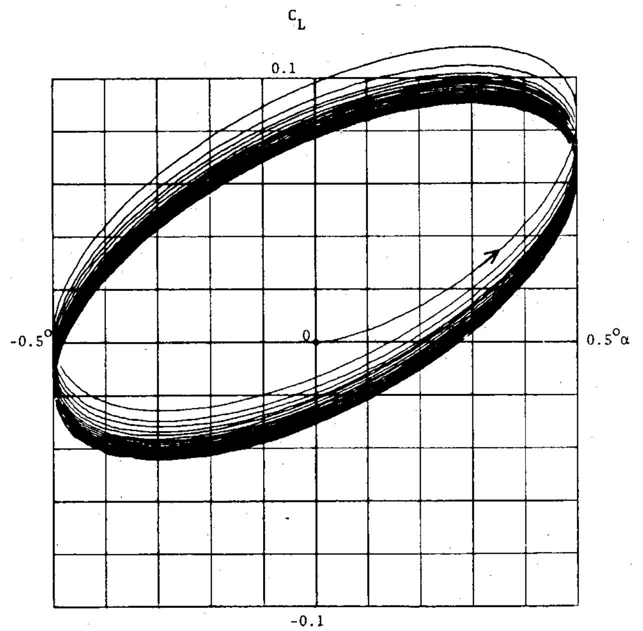

Fig. 2 NACA 64A006 airfoil, $M_\infty = 0.9$, $k = 0.2$.

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